PROPOSED AMENDMENTS TO THE CLAIMS



Claims 1-18 (Canceled)

19. (Currently Amended) A method of analyzing and computing ~~modelling~~ anisotropic turbulent flow of ~~in~~ an anisotropic fluid comprising:

providing input to a general purpose computer defining, for an anisotropic fluid, a set of moment equations governing time average thermal and turbulent motion, directional kinetic energy, shear, directional kinetic energy fluxes, and structure correlations;

instructing the general purpose computer to calculate $n^{th}$ order, wherein $n$ is odd, directional kinetic energy fluxes and structure correlation equations ~~closure relationships~~ using $(n + 1)^{th}$ order density gradient independent time average thermal and turbulent moment closure relationships to yield a set of closed time average turbulent moment equations;

using the set of closed time average turbulent moment equations to predict anisotropic turbulent flow of the anisotropic fluid; and

calculating and displaying a turbulent flow quantity of physical importance;

wherein the set of moment equations governing time average turbulent directional kinetic energy, shear, directional kinetic energy fluxes, and structure correlations is defined by:

Directional Kinetic ~~Turbulent~~ Energy

$$\frac{\partial}{\partial t}\left[\overline{u_1'^2}\right] + \overline{u_1}\frac{\partial}{\partial x_1}\left[\overline{u_1'^2}\right] + \overline{u_2}\frac{\partial}{\partial x_2}\left[\overline{u_1'^2}\right] + \overline{u_3}\frac{\partial}{\partial x_3}\left[\overline{u_1'^2}\right]$$
$$+ 2\left[\overline{u_1'^2}\frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_1'u_2'}\frac{\partial \overline{u_1}}{\partial x_2} + \overline{u_1'u_3'}\frac{\partial \overline{u_1}}{\partial x_3}\right]$$
$$+ \frac{1}{\overline{\rho}}\left[\frac{\partial}{\partial x_1}\left[\overline{\rho u_1' u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{\rho u_2' u_1'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{\rho u_3' u_1'^2}\right]\right]$$
$$= 0$$

~~Turbulent~~ Shear

$$\frac{\partial}{\partial t}\left[\overline{u_1'u_2'}\right] + \overline{u_1}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right] + \overline{u_2}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right] + \overline{u_3}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'}\right]$$
$$+ \overline{u_1'u_2'}\frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_2'^2}\frac{\partial \overline{u_1}}{\partial x_2} + \overline{u_2'u_3'}\frac{\partial \overline{u_1}}{\partial x_3}$$
$$+ \overline{u_1'^2}\frac{\partial \overline{u_2}}{\partial x_1} + \overline{u_1'u_2'}\frac{\partial \overline{u_2}}{\partial x_2} + \overline{u_1'u_3'}\frac{\partial \overline{u_2}}{\partial x_3}$$
$$+ \frac{1}{\overline{\rho}}\left[\frac{\partial}{\partial x_1}\left[\overline{\rho u_2' u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{\rho u_1' u_2'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{\rho u_1' u_2' u_3'}\right]\right]$$
$$= 0$$

Directional Kinetic ~~Turbulent~~ Energy Fluxes

$$\frac{\partial}{\partial t}\left[\overline{u_1' u_1'^2}\right] + \overline{u_1}\frac{\partial}{\partial x_1}\left[\overline{u_1' u_1'^2}\right] + \overline{u_2}\frac{\partial}{\partial x_2}\left[\overline{u_1' u_1'^2}\right] + \overline{u_3}\frac{\partial}{\partial x_3}\left[\overline{u_1' u_1'^2}\right]$$

$$+ 3\left[\overline{u_1' u_1'^2}\frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_2' u_1'^2}\frac{\partial \overline{u_1}}{\partial x_2} + \overline{u_3' u_1'^2}\frac{\partial \overline{u_1}}{\partial x_3}\right]$$

$$- 3\,\overline{u_1'^2}\left[\frac{\partial}{\partial x_1}\left[\overline{u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_1' u_2'}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1' u_3'}\right]\right]$$

$$+ 3\,\overline{\overline{c_1^2}}\left[\frac{\delta}{\delta x_1}\left[\overline{u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_1' u_2'}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1' u_3'}\right]\right]$$

$$+ 3\left[\overline{u_1'^2}\frac{\partial}{\partial x_1}\left[\overline{\overline{c_1^2}}\right] + \overline{u_1' u_2'}\frac{\partial}{\partial x_2}\left[\overline{\overline{c_1^2}}\right] + \overline{u_1' u_3'}\frac{\partial}{\partial x_3}\left[\overline{\overline{c_1^2}}\right]\right]$$

$$+ \frac{\partial}{\partial x_1}\left[\overline{u_1'^2 u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_1' u_2' u_1'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1' u_3' u_1'^2}\right]$$

$$+ \left[\overline{u_1'^2 u_1'^2} - 3\,\overline{u_1'^2}\left[\overline{u_1'^2}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial \overline{\rho}}{\partial x_1}$$

$$+ \left[\overline{u_1' u_2' u_1'^2} - 3\,\overline{u_1' u_2'}\left[\overline{u_1'^2}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial \overline{\rho}}{\partial x_2}$$

$$+ \left[\overline{u_1' u_3' u_1'^2} - 3\,\overline{u_1' u_3'}\left[\overline{u_1'^2}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial \overline{\rho}}{\partial x_3}$$

$$= 0$$

and

~~Directional Turbulent Energy Fluxes~~

$$\frac{\partial}{\partial t}\left[\overline{u_1'u_2'^2}\right]+\overline{u_2}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'^2}\right]+\overline{u_1}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'^2}\right]+\overline{u_3}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'^2}\right]$$

$$+\ 2\left[\overline{u_1'u_2'^2}\frac{\partial\overline{u_2}}{\partial x_2}+\overline{u_2'u_1'^2}\frac{\partial\overline{u_2}}{\partial x_1}+\overline{u_1'u_2'u_3'}\frac{\partial\overline{u_2}}{\partial x_3}\right]$$

$$+\ \overline{u_2'u_2'^2}\frac{\partial\overline{u_1}}{\partial x_2}+\overline{u_1'u_1'^2}\frac{\partial\overline{u_1}}{\partial x_1}+\overline{u_3'u_1'^2}\frac{\partial\overline{u_1}}{\partial x_3}$$

$$-\ 2\,\overline{u_1'u_2'}\left[\frac{\partial}{\partial x_1}\left[\overline{u_2'^2}\right]+\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right]+\frac{\partial}{\partial x_3}\left[\overline{u_2'u_3'}\right]\right]$$

$$-\ \overline{u_2'^2}\left[\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right]+\frac{\partial}{\partial x_1}\left[\overline{u_1'^2}\right]+\frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'}\right]\right]$$

$$+\ 2\left[\overline{\overline{c_2^2}}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right]+\overline{\overline{c_1c_2}}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right]+\overline{\overline{c_2c_3}}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'}\right]\right]$$

$$+\ \overline{\overline{c_1c_2}}\frac{\partial}{\partial x_2}\left[\overline{u_2'^2}\right]+\overline{\overline{c_1^2}}\frac{\partial}{\partial x_1}\left[\overline{u_2'^2}\right]+\overline{\overline{c_1c_3}}\frac{\partial}{\partial x_3}\left[\overline{u_2'^2}\right]$$

$$+\ 2\left[\overline{u_2'^2}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right]+\overline{u_1'u_2'}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right]+\overline{u_2'u_3'}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'}\right]\right]$$

$$+\ \overline{u_1'u_2'}\frac{\partial}{\partial x_2}\left[\overline{\overline{c_2^2}}\right]+\overline{u_1'^2}\frac{\partial}{\partial x_1}\left[\overline{\overline{c_2^2}}\right]+\overline{u_1'u_3'}\frac{\partial}{\partial x_3}\left[\overline{\overline{c_1^2}}\right]$$

$$+\ \frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'u_2'^2}\right]+\frac{\partial}{\partial x_1}\left[\overline{u_1'^2u_2'^2}\right]+\frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'u_2'^2}\right]$$

$$+\left[\overline{u_1'u_2'u_2'^2}-3\,\overline{u_1'u_2'}\left[\overline{u_2'^2}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial\overline{\rho}}{\partial x_2}$$

$$+\left[\overline{u_1'^2u_2'^2}-\overline{u_1'^2}\left[\overline{u_2'^2}\right]-2\,\overline{u_1'u_2'}\left[\overline{u_1'u_2'}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial\overline{\rho}}{\partial x_1}$$

$$+\left[\overline{u_1'u_3'u_2'^2}-\overline{u_1'u_3'}\left[\overline{u_2'^2}\right]-2\,\overline{u_1'u_2'}\left[\overline{u_2'u_3'}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial\overline{\rho}}{\partial x_3}$$

$$=0$$

Structure Correlations

$$\frac{\partial}{\partial t}\left[\overline{u_1'u_2'u_3'}\right]+\overline{u_1}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'u_3'}\right]+\overline{u_2}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'u_3'}\right]+\overline{u_3}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'u_3'}\right]$$
$$+\overline{u_1'u_2'u_3'}\frac{\partial\overline{u_1}}{\partial x_1}+\overline{u_3'u_2'^2}\frac{\partial\overline{u_2}}{\partial x_2}+\overline{u_2'u_3'^2}\frac{\partial\overline{u_1}}{\partial x_3}$$
$$+\overline{u_3'u_1'^2}\frac{\partial\overline{u_2}}{\partial x_1}+\overline{u_1'u_2'u_3'}\frac{\partial\overline{u_2}}{\partial x_2}+\overline{u_1'u_3'^2}\frac{\partial\overline{u_2}}{\partial x_3}$$
$$+\overline{u_2'u_1'^2}\frac{\partial\overline{u_3}}{\partial x_1}+\overline{u_1'u_2'^2}\frac{\partial\overline{u_3}}{\partial x_3}+\overline{u_1'u_2'u_3'}\frac{\partial\overline{u_3}}{\partial x_3}$$
$$-\overline{u_1'u_2'}\left[\frac{\partial}{\partial x_1}\left[\overline{u_1'u_3'}\right]+\frac{\partial}{\partial x_2}\left[\overline{u_2'u_3'}\right]+\frac{\partial}{\partial x_3}\left[\overline{u_3'^2}\right]\right]$$
$$-\overline{u_1'u_3'}\left[\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right]+\frac{\partial}{\partial x_2}\left[\overline{u_2'^2}\right]+\frac{\partial}{\partial x_3}\left[\overline{u_2'u_3'}\right]\right]$$
$$-\overline{u_2'u_3'}\left[\frac{\partial}{\partial x_1}\left[\overline{u_1'^2}\right]+\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right]+\frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'}\right]\right]$$
$$+\overline{u_1'^2}\frac{\partial}{\partial x_1}\left[\overline{\overline{c_2c_3}}\right]+\overline{u_1'u_2'}\frac{\partial}{\partial x_2}\left[\overline{\overline{c_2c_3}}\right]+\overline{u_1'u_3'}\frac{\partial}{\partial x_3}\left[\overline{\overline{c_2c_3}}\right]$$
$$+\overline{u_1'u_2'}\frac{\partial}{\partial x_1}\left[\overline{\overline{c_1c_3}}\right]+\overline{u_2'^2}\frac{\partial}{\partial x_2}\left[\overline{\overline{c_1c_3}}\right]+\overline{u_2'u_3'}\frac{\partial}{\partial x_3}\left[\overline{\overline{c_1c_3}}\right]$$
$$+\overline{u_1'u_3'}\frac{\partial}{\partial x_1}\left[\overline{\overline{c_1c_2}}\right]+\overline{u_2'u_3'}\frac{\partial}{\partial x_2}\left[\overline{\overline{c_1c_2}}\right]+\overline{u_3'^2}\,d\frac{\partial}{\partial x_3}\left[\overline{\overline{c_1c_2}}\right]$$
$$+\overline{\overline{c_1^2}}\frac{\partial}{\partial x_1}\left[\overline{u_2'u_3'}\right]+\overline{\overline{c_1c_2}}\frac{\partial}{\partial x_2}\left[\overline{u_2'u_3'}\right]+\overline{\overline{c_1c_3}}\frac{\partial}{\partial x_3}\left[\overline{u_2'u_3'}\right]$$
$$+\overline{\overline{c_1c_3}}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right]+\overline{\overline{c_2c_3}}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right]+\overline{\overline{c_3^2}}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'}\right]$$
$$+\overline{\overline{c_1c_2}}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_3'}\right]+\overline{\overline{c_2^2}}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_3'}\right]+\overline{\overline{c_2c_3}}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'}\right]$$
$$+\frac{\partial}{\partial x_1}\left[\overline{u_2'u_3'u_1'^2}\right]+\frac{\partial}{\partial x_2}\left[\overline{u_1'u_3'u_2'^2}\right]+\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'u_3'^2}\right]$$
$$+\left[\overline{u_2'u_3'u_1'^2}-\overline{u_2'u_3'}\left[\overline{u_1'^2}\right]-2\,\overline{u_1'u_2'}\left[\overline{u_1'u_3'}\right]\right]\overline{\overline{\rho}}\frac{1}{\overline{\rho}}\frac{\partial\overline{\rho}}{\partial x_1}$$
$$+\left[\overline{u_1'u_3'u_2'^2}-\overline{u_1'u_3'}\left[\overline{u_2'^2}\right]-2\,\overline{u_1'u_2'}\left[\overline{u_2'u_3'}\right]\right]\overline{\overline{\rho}}\frac{1}{\overline{\rho}}\frac{\partial\overline{\rho}}{\partial x_2}$$
$$+\left[\overline{u_1'u_2'u_3'^2}-\overline{u_1'u_2'}\left[\overline{u_3'^2}\right]-2\,\overline{u_1'u_3'}\left[\overline{u_2'u_3'}\right]\right]\overline{\overline{\rho}}\frac{1}{\overline{\rho}}\frac{\partial\overline{\rho}}{\partial x_3}$$
$$=0$$

20. (Currently Amended) A computer readable storage medium containing a set of instructions for a general purpose computer, the set of instructions defining a method of deriving a set of closed time average turbulent moment equations for analyzing and computing~~modelling~~ anisotropic turbulent flow of~~in~~ an anisotropic fluid comprising:

defining, for an anisotropic fluid, a set of moment equations governing time average thermal and turbulent motion, directional kinetic energy, shear, directional kinetic energy fluxes, and structure correlations;

calculating $n^{th}$ order , wherein $n$ is odd, directional kinetic energy fluxes and structure correlation equations ~~closure relationships~~ using $(n+1)^{th}$ order density gradient independent time average thermal and turbulent moment closure relationships to yield a set of closed time average turbulent moment equations;

using the set of closed time average turbulent moment equations to predict anisotropic turbulent flow of the anisotropic fluid; and

calculating and displaying a turbulent flow quantity of physical importance;

wherein the set of moment equations governing time average turbulent directional kinetic energy, shear, directional kinetic energy fluxes, and structure correlations is defined by:

Directional Kinetic ~~Turbulent~~ Energy

$$\frac{\partial}{\partial t}\left[\overline{u_1'^2}\right]+\overline{u_1}\frac{\partial}{\partial x_1}\left[\overline{u_1'^2}\right]+\overline{u_2}\frac{\partial}{\partial x_2}\left[\overline{u_1'^2}\right]+\overline{u_3}\frac{\partial}{\partial x_3}\left[\overline{u_1'^2}\right]$$
$$+\ 2\left[\overline{u_1'^2}\frac{\partial \overline{u_1}}{\partial x_1}+\overline{u_1'u_2'}\frac{\partial \overline{u_1}}{\partial x_2}+\overline{u_1'u_3'}\frac{\partial \overline{u_1}}{\partial x_3}\right]$$
$$+\ \frac{1}{\overline{\rho}}\left[\frac{\partial}{\partial x_1}\left[\overline{\rho u_1'u_1'^2}\right]+\frac{\partial}{\partial x_2}\left[\overline{\rho u_2'u_1'^2}\right]+\frac{\partial}{\partial x_3}\left[\overline{\rho u_3'u_1'^2}\right]\right]$$
$$=\ 0$$

~~Turbulent~~ Shear

$$\frac{\partial}{\partial t}\left[\overline{u_1'u_2'}\right] + \overline{u_1}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right] + \overline{u_2}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right] + \overline{u_3}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'}\right]$$
$$+ \overline{u_1'u_2'}\frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_2'^2}\frac{\partial \overline{u_1}}{\partial x_2} + \overline{u_2'u_3'}\frac{\partial \overline{u_1}}{\partial x_3}$$
$$+ \overline{u_1'^2}\frac{\partial \overline{u_2}}{\partial x_1} + \overline{u_1'u_2'}\frac{\partial \overline{u_2}}{\partial x_2} + \overline{u_1'u_3'}\frac{\partial \overline{u_2}}{\partial x_3}$$
$$+ \frac{1}{\overline{\rho}}\left[\frac{\partial}{\partial x_1}\left[\overline{\rho u_2'u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{\rho u_1'u_2'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{\rho u_1'u_2'u_3'}\right]\right]$$
$$= 0$$

Directional <u>Kinetic</u> ~~Turbulent~~ Energy Fluxes

$$\frac{\partial}{\partial t}\left[\overline{u_1'u_1'^2}\right] + \overline{u_1}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_1'^2}\right] + \overline{u_2}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_1'^2}\right] + \overline{u_3}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_1'^2}\right]$$
$$+ 3\left[\overline{u_1'u_1'^2}\frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_2'u_1'^2}\frac{\partial \overline{u_1}}{\partial x_2} + \overline{u_3'u_1'^2}\frac{\partial \overline{u_1}}{\partial x_3}\right]$$
$$- 3\,\overline{u_1'^2}\left[\frac{\partial}{\partial x_1}\left[\overline{u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'}\right]\right]$$
$$+ 3\,\overline{\overline{c_1^2}}\left[\frac{\delta}{\delta x_1}\left[\overline{u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'}\right]\right]$$
$$+ 3\left[\overline{u_1'^2}\frac{\partial}{\partial x_1}\left[\overline{\overline{c_1^2}}\right] + \overline{u_1'u_2'}\frac{\partial}{\partial x_2}\left[\overline{\overline{c_1^2}}\right] + \overline{u_1'u_3'}\frac{\partial}{\partial x_3}\left[\overline{\overline{c_1^2}}\right]\right]$$
$$+ \frac{\partial}{\partial x_1}\left[\overline{u_1'^2u_1'^2}\right] + \frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'u_1'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'u_1'^2}\right]$$
$$+ \left[\overline{u_1'^2u_1'^2} - 3\,\overline{u_1'^2}\left[\overline{u_1'^2}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial \overline{\rho}}{\partial x_1}$$
$$+ \left[\overline{u_1'u_2'u_1'^2} - 3\,\overline{u_1'u_2'}\left[\overline{u_1'^2}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial \overline{\rho}}{\partial x_2}$$
$$+ \left[\overline{u_1'u_3'u_1'^2} - 3\,\overline{u_1'u_3'}\left[\overline{u_1'^2}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial \overline{\rho}}{\partial x_3}$$
$$= 0$$

<u>and</u>

~~Directional Turbulent Energy Fluxes~~

$$\frac{\partial}{\partial t}\left[\overline{u_1' u_2'^2}\right] + \overline{u_2}\frac{\partial}{\partial x_2}\left[\overline{u_1' u_2'^2}\right] + \overline{u_1}\frac{\partial}{\partial x_1}\left[\overline{u_1' u_2'^2}\right] + \overline{u_3}\frac{\partial}{\partial x_3}\left[\overline{u_1' u_2'^2}\right]$$

$$+ \; 2\left[\overline{u_1' u_2'^2}\frac{\partial \overline{u_2}}{\partial x_2} + \overline{u_2' u_1'^2}\frac{\partial \overline{u_2}}{\partial x_1} + \overline{u_1' u_2' u_3'}\frac{\partial \overline{u_2}}{\partial x_3}\right]$$

$$+ \; \overline{u_2' u_2'^2}\frac{\partial \overline{u_1}}{\partial x_2} + \overline{u_1' u_1'^2}\frac{\partial \overline{u_1}}{\partial x_1} + \overline{u_3' u_1'^2}\frac{\partial \overline{u_1}}{\partial x_3}$$

$$- \; 2\,\overline{u_1' u_2'}\left[\frac{\partial}{\partial x_1}\left[\overline{u_2'^2}\right] + \frac{\partial}{\partial x_1}\left[\overline{u_1' u_2'}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_2' u_3'}\right]\right]$$

$$- \; \overline{u_2'^2}\left[\frac{\partial}{\partial x_2}\left[\overline{u_1' u_2'}\right] + \frac{\partial}{\partial x_1}\left[\overline{u_1'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1' u_3'}\right]\right]$$

$$+ \; 2\left[\overline{\overline{c_2^2}}\frac{\partial}{\partial x_1}\left[\overline{u_1' u_2'}\right] + \overline{\overline{c_1 c_2}}\frac{\partial}{\partial x_1}\left[\overline{u_1' u_2'}\right] + \overline{\overline{c_2 c_3}}\frac{\partial}{\partial x_3}\left[\overline{u_1' u_2'}\right]\right]$$

$$+ \; \overline{\overline{c_1 c_2}}\frac{\partial}{\partial x_2}\left[\overline{u_2'^2}\right] + \overline{\overline{c_1^2}}\frac{\partial}{\partial x_1}\left[\overline{u_2'^2}\right] + \overline{\overline{c_1 c_3}}\frac{\partial}{\partial x_3}\left[\overline{u_2'^2}\right]$$

$$+ \; 2\left[\overline{u_2'^2}\frac{\partial}{\partial x_2}\left[\overline{u_1' u_2'}\right] + \overline{u_1' u_2'}\frac{\partial}{\partial x_1}\left[\overline{u_1' u_2'}\right] + \overline{u_2' u_3'}\frac{\partial}{\partial x_3}\left[\overline{u_1' u_2'}\right]\right]$$

$$+ \; \overline{u_1' u_2'}\frac{\partial}{\partial x_2}\left[\overline{\overline{c_2^2}}\right] + \overline{u_1'^2}\frac{\partial}{\partial x_1}\left[\overline{\overline{c_2^2}}\right] + \overline{u_1' u_3'}\frac{\partial}{\partial x_3}\left[\overline{\overline{c_1^2}}\right]$$

$$+ \; \frac{\partial}{\partial x_2}\left[\overline{u_1' u_2' u_2'^2}\right] + \frac{\partial}{\partial x_1}\left[\overline{u_1'^2 u_2'^2}\right] + \frac{\partial}{\partial x_3}\left[\overline{u_1' u_3' u_2'^2}\right]$$

$$+ \; \left[\overline{u_1' u_2' u_2'^2} - 3\,\overline{u_1' u_2'}\left[\overline{u_2'^2}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial \overline{\rho}}{\partial x_2}$$

$$+ \; \left[\overline{u_1'^2 u_2'^2} - \overline{u_1'^2}\left[\overline{u_2'^2}\right] - 2\,\overline{u_1' u_2'}\left[\overline{u_1' u_2'}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial \overline{\rho}}{\partial x_1}$$

$$+ \; \left[\overline{u_1' u_3' u_2'^2} - \overline{u_1' u_3'}\left[\overline{u_2'^2}\right] - 2\,\overline{u_1' u_2'}\left[\overline{u_2' u_3'}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial \overline{\rho}}{\partial x_3}$$

$$= 0$$

Structure Correlations

$$\frac{\partial}{\partial t}\left[\overline{u_1'u_2'u_3'}\right]+\overline{u_1}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'u_3'}\right]+\overline{u_2}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'u_3'}\right]+\overline{u_3}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'u_3'}\right]$$
$$+\overline{u_1'u_2'u_3'}\frac{\partial\overline{u_1}}{\partial x_1}+\overline{u_3'u_2'^2}\frac{\partial\overline{u_2}}{\partial x_2}+\overline{u_2'u_3'^2}\frac{\partial\overline{u_1}}{\partial x_3}$$
$$+\overline{u_3'u_1'^2}\frac{\partial\overline{u_2}}{\partial x_1}+\overline{u_1'u_2'u_3'}\frac{\partial\overline{u_2}}{\partial x_2}+\overline{u_1'u_3'^2}\frac{\partial\overline{u_2}}{\partial x_3}$$
$$+\overline{u_2'u_1'^2}\frac{\partial\overline{u_3}}{\partial x_1}+\overline{u_1'u_2'^2}\frac{\partial\overline{u_2}}{\partial x_3}+\overline{u_1'u_2'u_3'}\frac{\partial\overline{u_3}}{\partial x_3}$$
$$-\overline{u_1'u_2'}\left[\frac{\partial}{\partial x_1}\left[\overline{u_1'u_3'}\right]+\frac{\partial}{\partial x_2}\left[\overline{u_2'u_3'}\right]+\frac{\partial}{\partial x_3}\left[\overline{u_3'^2}\right]\right]$$
$$-\overline{u_1'u_3'}\left[\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right]+\frac{\partial}{\partial x_2}\left[\overline{u_2'^2}\right]+\frac{\partial}{\partial x_3}\left[\overline{u_2'u_3'}\right]\right]$$
$$-\overline{u_2'u_3'}\left[\frac{\partial}{\partial x_1}\left[\overline{u_1'^2}\right]+\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right]+\frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'}\right]\right]$$
$$+\overline{u_1'^2}\frac{\partial}{\partial x_1}\left[\overline{\overline{c_2c_3}}\right]+\overline{u_1'u_2'}\frac{\partial}{\partial x_2}\left[\overline{\overline{c_2c_3}}\right]+\overline{u_1'u_3'}\frac{\partial}{\partial x_3}\left[\overline{\overline{c_2c_3}}\right]$$
$$+\overline{u_1'u_2'}\frac{\partial}{\partial x_1}\left[\overline{\overline{c_1c_3}}\right]+\overline{u_2'^2}\frac{\partial}{\partial x_2}\left[\overline{\overline{c_1c_3}}\right]+\overline{u_2'u_3'}\frac{\partial}{\partial x_3}\left[\overline{\overline{c_1c_3}}\right]$$
$$+\overline{u_1'u_3'}\frac{\partial}{\partial x_1}\left[\overline{\overline{c_1c_2}}\right]+\overline{u_2'u_3'}\frac{\partial}{\partial x_2}\left[\overline{\overline{c_1c_2}}\right]+\overline{u_3'^2}\mathrm{d}\frac{\partial}{\partial x_3}\left[\overline{\overline{c_1c_2}}\right]$$
$$+\overline{\overline{c_1'}}\frac{\partial}{\partial x_1}\left[\overline{u_2'u_3'}\right]+\overline{\overline{c_1c_2}}\frac{\partial}{\partial x_2}\left[\overline{u_2'u_3'}\right]+\overline{\overline{c_1c_3}}\frac{\partial}{\partial x_3}\left[\overline{u_2'u_3'}\right]$$
$$+\overline{\overline{c_1c_3}}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_2'}\right]+\overline{\overline{c_2c_3}}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_2'}\right]+\overline{\overline{c_3'}}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'}\right]$$
$$+\overline{\overline{c_1c_2}}\frac{\partial}{\partial x_1}\left[\overline{u_1'u_3'}\right]+\overline{\overline{c_2'}}\frac{\partial}{\partial x_2}\left[\overline{u_1'u_3'}\right]+\overline{\overline{c_2c_3}}\frac{\partial}{\partial x_3}\left[\overline{u_1'u_3'}\right]$$
$$+\frac{\partial}{\partial x_1}\left[\overline{u_2'u_3'u_1'^2}\right]+\frac{\partial}{\partial x_2}\left[\overline{u_1'u_3'u_2'^2}\right]+\frac{\partial}{\partial x_3}\left[\overline{u_1'u_2'u_3'^2}\right]$$
$$+\left[\overline{u_2'u_3'u_1'^2}-\overline{u_2'u_3'}\left[\overline{u_1'^2}\right]-2\,\overline{u_1'u_2'}\left[\overline{u_1'u_3'}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial\overline{\rho}}{\partial x_1}$$
$$+\left[\overline{u_1'u_3'u_2'^2}-\overline{u_1'u_3'}\left[\overline{u_2'^2}\right]-2\,\overline{u_1'u_2'}\left[\overline{u_2'u_3'}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial\overline{\rho}}{\partial x_2}$$
$$+\left[\overline{u_1'u_2'u_3'^2}-\overline{u_1'u_2'}\left[\overline{u_3'^2}\right]-2\,\overline{u_1'u_3'}\left[\overline{u_2'u_3'}\right]\right]\frac{1}{\overline{\rho}}\frac{\partial\overline{\rho}}{\partial x_3}$$
$$=0$$